Licenciatura em Gestão

Operational Research Chapter 4

2018-2019

100 1911-2011 ANOS

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4. The Transportation and the Assignment Problems

4.1 Introduction

4.2 Transportation Problem

4.3 Assignment Problem

The Transportation Problem

Transportation Problem (TP) — determine the quantities of an homogeneous commodity to be shipped from a set of **distribution centres** — the origins (sources) — to a set of **receiving centres** — the destinations — such that the total cost is minimised.

Applications:

- Products transportation;
- Production planning;
- Scheduling human resources;
- ...

The Transportation Problem

Data: m = number of origins; s_i = **supply** at origin i;

n = number of destinations; d_j = **demand** at destination j;

c_{ij} = cost per unit distributed from i to j

Assumption: the TP is **balanced**, that is, the total supply and total demand are equal.

x_{ij} – amount to ship from origin i to destination j.

Z – total cost of the transportation plan

LP formulation:

$$\text{Minimize } Z=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = s_i \quad (i=1,\ldots,m) \\ \sum_{i=1}^{m} x_{ij} = d_j \quad (j=1,\ldots,n) \\ x_{ij} \geq 0 \ (i=1,\ldots,m;\ j=1,\ldots,n) \end{cases}$$

TP Prototype Example

	A	B	C	D	E	F	G	H	I	J
1	P&T Co. Distribution Problem									
2										
3					shipping cost (m.u./per truck load)					
4					dest ination					
5				W1	W2	W3	W4	supply		
6			C1	464	513	654	867	75		
7		origin	C2	352	416	690	791	125		
8			C3	995	682	388	685	100		
9			demand	80	65	70	85			300
10										300
11										
12					solution (quant. to be shipped)					
13					dest ination					
14				W1	W2	W3	W4	total		supply
15			C1	0	0	0	0	0	=	75
16		origin	C2	0	0	0	0	0	=	125
17			C3	0	0	0	0	0	=	100
18			total	0	0	0	0	0	=	COST
19				=	=	=	=			
20			demand	80	65	70	85			
21										

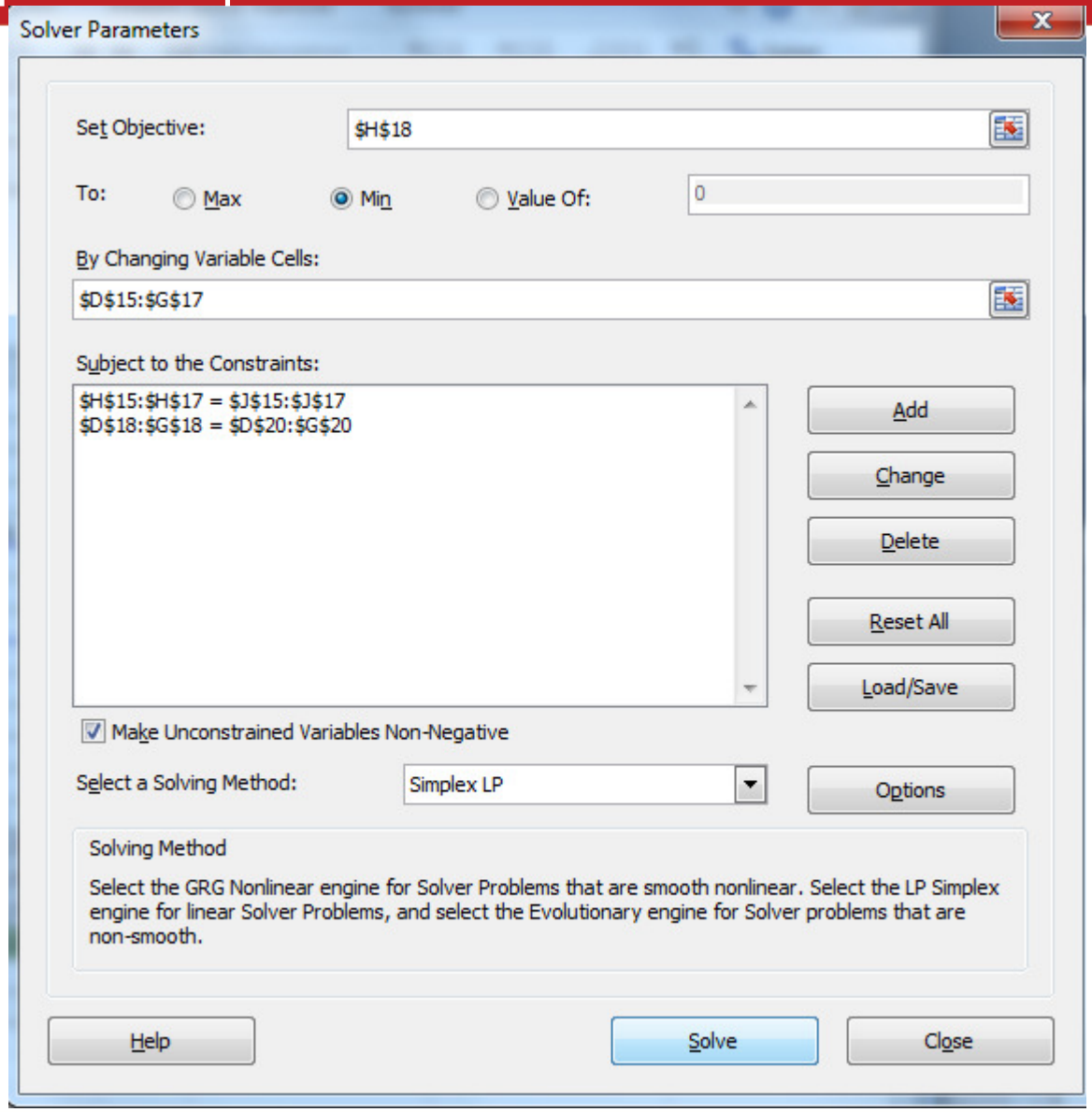
J9=SUM(H6:H8)
J10=SUM(D9:G9)

H15=SUM(D15:G15)
H16=SUM(D16:G16)
H17=SUM(D17:G17)

H18=SUMPRODUCT(D6:G8;D15:G17)

D18=SUM(D15:D17) E18=SUM(E15:E17) F18=SUM(F15:F17) G18=SUM(G15:G17)

TP Prototype Example



TP Prototype Example reports

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Microsoft Excel 8.0e Answer Report
Worksheet: [PT_prot_P&T.xls]Sheet1
Report Created: 29-04-2003 20:06:05

Target Cell (Min)

Cell	Name	Original Value	Final Value
$H$18	total total	0	152535

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$15	F1 A1	0	0
$E$15	F1 A2	0	20
$F$15	F1 A3	0	0
$G$15	F1 A4	0	55
$D$16	F2 A1	0	80
$E$16	F2 A2	0	45
$F$16	F2 A3	0	0
$G$16	F2 A4	0	0
$D$17	F3 A1	0	0
$E$17	F3 A2	0	0
$F$17	F3 A3	0	70
$G$17	F3 A4	0	30

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$15	F1 total	75	$H$15=$J$15	Binding	0
$H$16	F2 total	125	$H$16=$J$16	Binding	0
$H$17	F3 total	100	$H$17=$J$17	Not Binding	0
$D$18	total A1	80	$D$18=$D$20	Not Binding	0
$E$18	total A2	65	$E$18=$E$20	Not Binding	0
$F$18	total A3	70	$F$18=$F$20	Not Binding	0
$G$18	total A4	85	$G$18=$G$20	Not Binding	0

Microsoft Excel 8.0e Sensitivity Report
Worksheet: [PT_prot_P&T.xls]Sheet1
Report Created: 29-04-2003 20:06:05

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$D$15	F1 A1	0	15	464	1E+30	15
$E$15	F1 A2	20	0	513	15	21
$F$15	F1 A3	0	84	654	1E+30	84
$G$15	F1 A4	55	0	867	21	351
$D$16	F2 A1	80	0	352	15	1E+30
$E$16	F2 A2	45	0	416	21	15
$F$16	F2 A3	0	217	690	1E+30	217
$G$16	F2 A4	0	21	791	1E+30	21
$D$17	F3 A1	0	728	995	1E+30	728
$E$17	F3 A2	0	351	682	1E+30	351
$F$17	F3 A3	70	0	388	84	1E+30
$G$17	F3 A4	30	0	685	351	84

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$H$15	F1 total	75	570	75	0	55
$H$16	F2 total	125	473	125	0	45
$H$17	F3 total	100	388	100	0	70
$D$18	total A1	80	-121	80	45	0
$E$18	total A2	65	-57	65	55	0
$F$18	total A3	70	0	70	0	1E+30
$G$18	total A4	85	297	85	70	0

The Transportation Problem

Properties of the TP

P1: The TP has at least one feasible solution.

Corollary: The TP has optimal solution.

P2: A TP where supplies and demands have integer values has at least one optimal solution with all variables integer valued.

The Transportation Problem - Variants

Problems that have the same structure of parameters but:

(V1) total supply > total demand: origin constraints "≤".

Opt. Sol. : Part of the supply is not transported

(V2) total supply < total demand: destination constraints "≤".

Opt. Sol. : Part of the demand is not satisfied

(V3) Destination requiring demand between a minimum and a maximum value:

2 constraints at the destination: "≤ maximum demand" and "≥ minimum demand".

(V4) Origin producing supply between a minimum and a maximum value: ≈ (V3)

(V5) Infeasible link: corresponding variable is set to zero.

(V6) Maximization problem: in solver/excel choose OF: Max. ⊙

The Assignment Problem

Assignment Problem (AP)

Assign ***n*** people to ***n*** tasks – each person to a task, each task to a person – minimising the total assignment cost.

Aplications:

- Assign people to tasks;
- Production planning (operations to machines; products to plants)
- ….

The Assignment Problem

Data: n = number of persons and tasks

c_{ij} = cost associated with assignee ***i*** performing task ***j*** (i,j=1,…,n)

$$x_{ij} = \begin{cases} 1 & \text{if person } i \text{ is assigned to job } j \\ 0 & \text{otherwise} \end{cases}$$

Z – total cost of the assignment plan

LP Formulation:

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = 1 \quad (i=1,\ldots,n) \\ \sum_{i=1}^{n} x_{ij} = 1 \quad (j=1,\ldots,n) \\ x_{ij} \geq 0 \quad (i,j=1,\ldots,n) \\ x_{ij} \text{ binary } (i,j=1,\ldots,n) \end{cases}$$

AP Prototype Example

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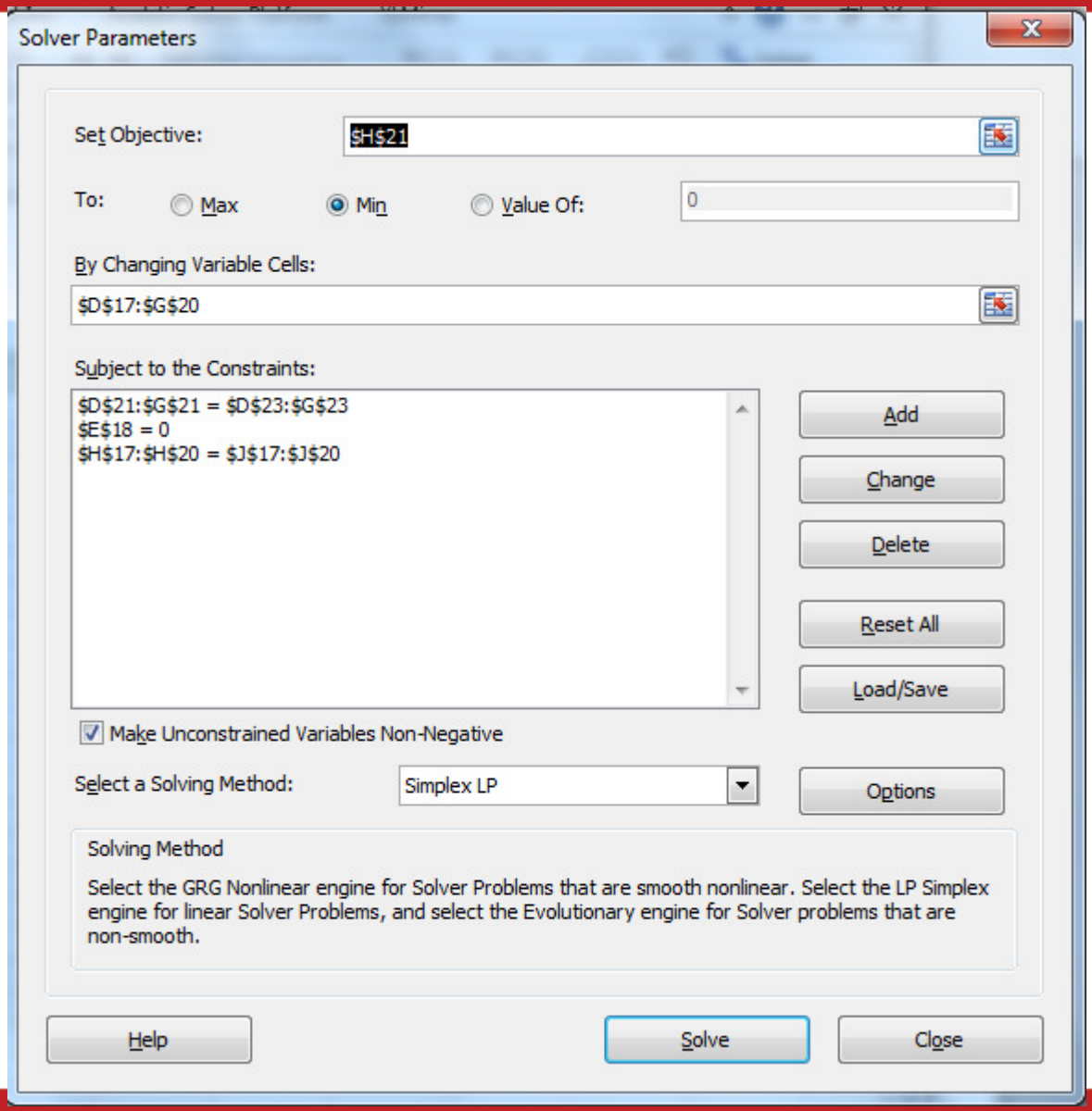
	A	B	C	D	E	F	G	H	I	J	K
1	' - JOB SHOP										
2											
3											
4				Cost ($) of materials handling							
5					locations						
6				L1	L2	L3	L4	si			
7			M1	13	16	12	11	1			
8		Machine	M2	15	-	13	20	1			
9			M3	5	7	10	6	1			
10			M4	12	20	15	13	1			
11			dj	1	1	1	1			4	
12										4	
13											
14											
15											
16				L1	L2	L3	L4			si	
17			M1	0	0	0	1	1	=	1	
18			M2	0	0	1	0	1	=	1	
19			M3	1	0	0	0	1	=	1	
20			M4	0	1	0	0	1	=	1	
21				1	1	1	1	29	=Z		
22				=	=	=	=				
23			dj	1	1	1	1				

H17=SUM(D17:G17)
H18=SUM(D18:G18)
H19=SUM(D19:G19)
H20=SUM(D20:G20)

H21=SUMPRODUCT(D7:G9;D17:G19)

D21=SUM(D17:D20) E21=SUM(E17:E20) F21=SUM(F17:F20) H21=SUM(G17:G20)

AP Prototype Example



AP Prototype Example

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Microsoft Excel 14.0 Answer Report

(…)

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$H$21	si	0	29

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$17	M1 L1	0	0	Contin
$E$17	M1 L2	0	0	Contin
$F$17	M1 L3	0	0	Contin
$G$17	M1 L4	0	1	Contin
$D$18	M2 L1	0	0	Contin
$E$18	M2 L2	0	0	Contin
$F$18	M2 L3	0	1	Contin
$G$18	M2 L4	0	0	Contin
$D$19	M3 L1	0	1	Contin
$E$19	M3 L2	0	0	Contin
$F$19	M3 L3	0	0	Contin
$G$19	M3 L4	0	0	Contin
$D$20	M4 L1	0	0	Contin
$E$20	M4 L2	0	1	Contin
$F$20	M4 L3	0	0	Contin
$G$20	M4 L4	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$21	L1	1	$D$21=$D$23	Binding	0
$E$21	L2	1	$E$21=$E$23	Binding	0
$F$21	L3	1	$F$21=$F$23	Binding	0
$G$21	L4	1	$G$21=$G$23	Binding	0
$H$17	M1 si	1	$H$17=$J$17	Binding	0
$H$18	M2 si	1	$H$18=$J$18	Binding	0
$H$19	M3 si	1	$H$19=$J$19	Binding	0
$H$20	M4 si	1	$H$20=$J$20	Binding	0
$E$18	M2 L2	0	$E$18=0	Binding	0

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [PA_JobShop.xls]Sheet1
Report Created: 10-11-2014 14:26:10

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$D$17	M1 L1	0	0	13	1	3
$E$17	M1 L2	0	1	16	1E+30	1
$F$17	M1 L3	0	0	12	3	1
$G$17	M1 L4	1	0	11	3	1E+30
$D$18	M2 L1	0	1	15	1E+30	1
$E$18	M2 L2	0	-16	0	1E+30	16
$F$18	M2 L3	1	0	13	1	1E+30
$G$18	M2 L4	0	8	20	1E+30	8
$D$19	M3 L1	1	0	5	2	1
$E$19	M3 L2	0	0	7	1	2
$F$19	M3 L3	0	6	10	1E+30	6
$G$19	M3 L4	0	3	6	1E+30	3
$D$20	M4 L1	0	2	0	1E+30	2
$E$20	M4 L2	1	0	0	2	1E+30
$F$20	M4 L3	0	3	0	1E+30	3
$G$20	M4 L4	0	4	0	1E+30	4

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$21	L1	1	5	1	0	1
$E$21	L2	1	7	1	0	0
$F$21	L3	1	4	1	0	0
$G$21	L4	1	3	1	0	1
$H$17	M1 si	1	8	1	1	0
$H$18	M2 si	1	9	1	0	0
$H$19	M3 si	1	0	1	0	1E+30
$H$20	M4 si	1	-7	1	0	0

The Assignment Problem - Variants

(V1) number of people > number of tasks: people constraints "≤".

Opt.Sol.: some people is not assigned

(V2) number of people < number of tasks : task constraints "≤".

OS: some tasks are not performed

(V3) some task can be assigned to more than one person

respective constraint " ≥**1** "

(V4) some person can perform more than one task ≈ (V3)

(V5) Infeasible links: person *i* cannot be assigned to task *j* then x_{ij}=0.

(V6) Maximization problem: in solver/excel choose OF TO: Max. ◉